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Efficient Demultiplexing Algorithm for Noncontiguous Carriers

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A channel separation algorithm for the frequency division multiple access/time division multiplexing (FDMA/TDM) scheme is presented. It is shown that implementation using this algorithm can be more effective than the fast Fourier transform (FFT) algorithm when only a small number of carriers need to be selected from many, such as satellite Earth terminals. The algorithm is based on polyphase filtering followed by application of a generalized Walsh-Hadamard transform (GWHT). Comparison of the transform technique used in this algorithm with discrete Fourier transform (DFT) and FFT is given. Estimates of the computational rates and power requirements to implement this system are also given.

Nomenclature

P_ρ = polyphase branch filter
W_K = twiddle factor
X_k = demultiplexed channel signal
x_ρ = input branch signal

Introduction

IN satellite communications systems incorporating small Earth stations, the application of multiple-access techniques of single channel per carrier (SCPC)/frequency division multiple access (FDMA) in the uplink, onboard switching and time division multiplexing (TDM) in the downlink is significantly effective in improving satellite transponder utilization and in reducing the required effective isotropic radiated power (EIRP) in both the Earth stations and the satellite.[1-3] The concept of FDMA/TDM can also be utilized in the Earth station applications, where digitally modulated carriers operating in FDMA mode are received and demodulated at a central station. The individually recovered carriers could then be routed, remodulated, and multiplexed onto a single carrier using time division. A coastal Earth station in maritime satellite communications[4,5] or a cluster of rural stations connected by terrestrial lines to a central station that receives satellite signals are examples of the above situation.

To perform the FDMA/TDM conversion, a multicarrier demodulator will be required at the central station. A conceptual block diagram of the multicarrier demodulator is shown in Fig. 1. The channel separation can be achieved digitally via a bank of digital filters with desired center frequencies, corresponding to the K carriers in the FDMA signal. After the channel separation, each carrier needs to be downconverted to baseband. The sampling rate of the downconverted channels will be with respect to the rate of the analog-to-digital (A/D) converter. In other words, the individual channels are obtained at a high sampling rate. Consequently, after the downconversion a signal-processing block is needed to provide a digital signal that is at the lowest permissible sampling rate consistent with preservation of the channel information. This can be achieved by decimating the A/D sampling rate to a new sampling rate (which is with respect to the individual channel bandwidth). The decimated channel signals are then passed through individual demodulator circuits to recover the message signal. The outputs of the demodulator can then be combined into a single TDM stream by time interleaving the samples of each channel.

The method of obtaining individual channels at baseband, suggested in the above discussion, is computationally inefficient if implemented directly. The bandpass filtering to obtain individual channels is performed on a signal that is at the rate of the sampler. This high-speed operation will require a large number of taps to implement the filters. Consequently, the computational rate and the power consumption for the filtering will be very high. The same can be said for the downconversion operation since it too is performed on the high sampling side. The decimation operation requires low-pass filtering of the signal first and then reducing the rate of the signal, which means the low-pass filtering is done at the high sampling side. Thus, the decimation too will require a high computational rate.

If the SCPC carriers are uniformly spaced, the sampling rate reduction, the channel separation, and the downconversion can be efficiently realized as one signal-processing block. This signal-processing block is known as a transmultiplexer (TMUX),[6,7] or a uniform channel filter bank.[8] The filter bank structure is implemented with the aid of a commutator, a bank of polyphase filters, and a discrete Fourier transform (DFT) implemented via a fast Fourier transform (FFT), which makes this algorithm computationally efficient.

In an Earth station application, it may be of interest to separate only a few carriers out of the hundreds received at the Earth station. This requires the separation of carriers that are noncontiguous (carriers that are not adjacent to each other), due to the arbitrary allocation of the desired carriers. The uniform channel filter bank is applicable in this situation, but the use of DFT in the filter bank algorithm will require a

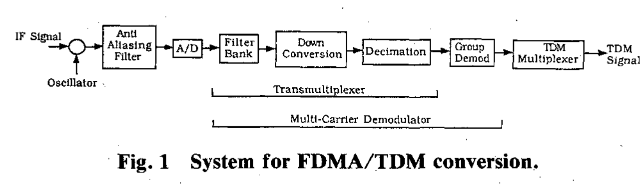

Fig. 1 System for FDMA/TDM conversion.

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prohibitively large number of computations as the number of desired carriers increase. On the other hand, the implementation of the DFT via FFT will waste computations by calculating undesired and unnecessary spectral components. This is because the structured FFT implementation will not allow any frequencies to be discarded until after all of the frequencies are computed. Thus, there is a need to develop a filter-bank algorithm that will enable processing of only the carriers of interest in an arbitrary and efficient manner.

Filter-Bank Structure for Noncontiguous Carriers

In this section, a computationally efficient demultiplexing algorithm for noncontiguous carriers is developed. The algorithm will enable the separation of any number of carriers in an arbitrary and efficient manner. It is derived from the uniform channel filter-bank structure and requires performing a generalized Walsh-Hadamard transform (GWHT)[9,10] rather than performing the DFT operation.

Conventional Polyphase Filter Approach

For the channel arrangement of Fig. 2, an analytical expression for the uniform channel filter bank can be written as[8,9]

$$X_k(m) = (-1)^m D\left\{W_K^{-\rho(1/2)}[P_\rho(m)W_K^{(1/2)mK} \times x_\rho(m)]\right\} \quad (1)$$

where

$k = 0,1,2,\ldots, K-1$
$\rho = 0,1,2,\ldots, K-1$
m = decimated sample number
K = number of channels
D = symbolic for DFT
$W_K = e^{(j2\pi)/K}$

The branch signal x_ρ and polyphase branch filter P_ρ are given by

$$x_\rho(m) = x(mM + \rho)$$

$$P_\rho(m) = h(mM - \rho)$$

where $h(n)$ is a low-pass finite impulse response (FIR) filter with cutoff equal to half of the channel spacing and its impulse response sampled at the rate of the composite signal. M is the decimation factor which is equal to K for this study. The block diagram of the filter-bank structure is shown in Fig. 3.

In this structure, the complex digital signal $x(n)$, operating at a sampling rate of F complex samples per second (2π in terms of normalized frequency), is first decimated by factor K and fed into the K branches using a clockwise commutator. Consequently, the K branch signals are at a sampling rate of F/K complex samples per second ($2\pi/K$ in terms of normalized frequency). The decimation operation will not alter the channel amplitude spectrum since for complex sampling, at integer multiples of the sampling rate, the aliased components will fall on top of the spectrum from 0 to F. Thus, the signal and the image components simply add up. After the commutation, the convolution of the ρth input signal and the ρth branch polyphase filter is performed. The polyphase filters are all-pass filters with different phase shift corresponding to each value of ρ. They introduce phase shifts in different frequency bands of the ρ branch signals. The polyphase filters are decimated versions of the low-pass FIR filter $h(n)$. If $h(n)$ has N taps, then each polyphase filter will have N/K taps. At the output of the polyphase filters, the branch signals are phase modified by the factor $W_K^{-\rho/2}$, for $\rho = 0,1,2,\ldots, K-1$. The outputs of the phase modifier are input to the DFT block. Different phase rotation in different frequency bands introduced by the polyphase filters are compensated by the DFT. The outputs of the DFT are the sums of the ρ inputs after getting phase shifted by multiples of $2\pi ab/K$, for $a,b = 0,1,2,\ldots, K-1$. At each output, the in-phase components

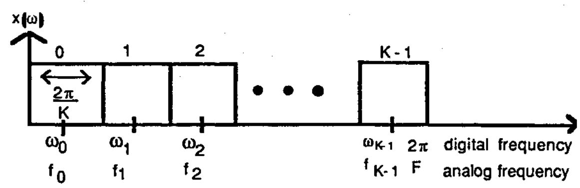

Fig. 2 Channel arrangements.

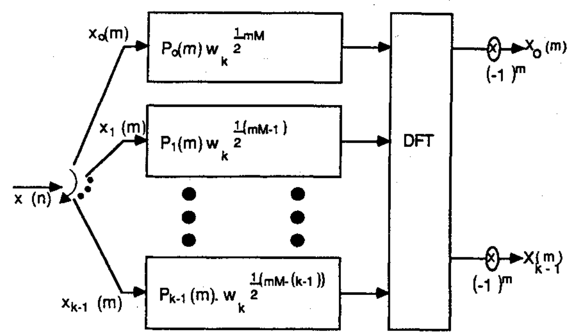

Fig. 3 Channel filter-bank structure.

of the resulting channels add, while the others cancel. The outputs of the DFT are modified by a multiplicative term of $(-1)^m$, which shifts the amplitude spectrum of the individual channels from the frequency offset of 1/2 (in normalized frequency) to 0.

Polyphase GWHT Approach

The above structure will be computationally inefficient for the Earth station application since the computation rate for the DFT will increase linearly with the number of desired carriers. The implementation of the DFT via FFT will waste computations since the desired carriers cannot be accessed until all of the carriers are processed.

A computationally efficient system can be achieved by replacing the DFT block in Eq. (1) by the GWHT. That is, replacing D by $(1/K)D\tilde{W}^{T*}\tilde{W}$ (where T denotes transpose and * denotes complex conjugate), we have

$$X_k(m) = (-1)^m(1/K)D\tilde{W}^{T*}(\tilde{W}\{W_K^{-\rho(1/2)}[P_\rho(m)W_K^{(1/2)mM} \times x_\rho(m)]\}) \quad (2)$$

where

$$\tilde{W}_{ab} = \exp\left[-j(2\pi/m)\sum_{i=0}^{n-1} a_i b_i\right]$$
$$a,b = 0,1,2,\ldots, NT-1 \quad (3)$$

where a_i and b_i are represented in the m-array representation as discussed in Ref. 10 and NT is the transform size. The block diagram of this system is shown in Fig. 4.

The substitution of $(1/K)D\tilde{W}^{T*}\tilde{W}$ for D will not change anything analytically since $\tilde{W}^{T*}\tilde{W} = KI$, where I is the identity matrix. The only difference now is that the transform is implemented in two stages. By denoting the input branch signals to the transform block by $y_\rho(m)$, it is observed that the vector $\tilde{W}\{y_\rho(m)\}$ can be recognized as the K-point GWHT of the sequence $y_\rho(m)$, for $\rho = 0,1,2,\ldots, M-1$. The coefficients of the $\tilde{W}$ matrix are highly dependent on the set of parameters m and n (in GWHT, $NT = m^n$, where NT is the transform size and m and n are integers). The transform size NT can either be equal to the number of channels K, or greater than K. For the case $NT = m^1$ ($n = 1$), $\tilde{W}$ is the DFT matrix and, at the other extreme, when $NT = 2^n$ ($m = 2$), $\tilde{W}$ reduces to the conventional Walsh-Hadamard matrix (that is, all of the coefficients of $\tilde{W}$ are ± 1).[10] In other cases depending on the parameters

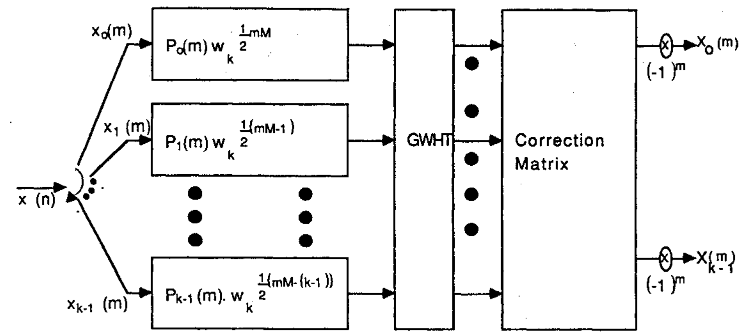

Fig. 4 Channel filter bank for noncontiguous carriers.

m and n, the coefficients of the $\tilde{W}$ matrix are either ± 1 or $\pm j$. Thus, by choosing appropriate values of m and n, the $\tilde{W}$ matrix will have trivial coefficients and a structured implementation of the K-point GWHT will result in zero multiplications.

After the K-point GWHT, the branch signals are processed by the operator $C = D\tilde{W}^{T*}$. The combined operation of $\tilde{W}$ and $D\tilde{W}^{T*}$ can be employed to determine the components of the desired carriers that are not necessarily adjacent to each other. The operator C can be thought of as performing correction on the signal components so that the results obtained by GWHT and C are the same as those obtained by DFT. Thus, after performing the K-point GWHT, we perform the correction on only the branch signals that represent the desired carriers. The input to the correction is K-point, but the output is only L-point, where L is the small number of carriers of interest. The correction matrix has a very simple structure (with many zeros), and this can be further exploited during implementation. The number of zero-valued coefficients in the correction matrix C is also dependent on m and n. The correction matrix can be computed off-line and stored. The coefficients can be retrieved from storage as the need arises. The computational savings obtained by this scheme over the DFT approach exist because the K-point GWHT results in zero multiplications and the correction is performed on only the carriers of interest.

Computational Rates and Power Requirements

This section presents estimates of the computational rates that are required to implement the polyphase filters, the preprocessor (the bank of multipliers before the transform), the proposed transform technique, and the postprocessor (the bank of multipliers at the output of the transform).

In Ref. 11, a survey of high speed and low power consumption devices is given. The power estimates presented in Ref. 11 are based on a 16 × 16-bit multiplier with a 25-ns multiplication time and a power dissipation of 150 mW. The power estimates presented here are based on the same multiplier. The parameters for the example of 800 channels are as follows:

Number of channels	= 800
Decimation factor	= 800
Modulation technique	= QPSK
Bit rate	= 64 kbps
Symbol rate	= 32 ksps
Channel spacing	= 45 kHz
Sampling frequency of the FDM signal	= 36 MHz
Sampling frequency of the branch signals	= 45 kHz
Number of taps in the prototype filter	= 7200
Number of taps in the polyphase filter	= 9
Transform size	= 1024

Computations for the Polyphase Filtering

The polyphase filters are implementated using the tapped delay-line structure with each filter consisting of nine taps with real coefficients. This results in nine real multiplies per sample. Since the filter coefficients of the FIR filter are real and the signal is complex (due to complex sampling), the filtering operation has to be performed separately on the real and the imaginary parts of the signal. Consequently, the number of real multiplies per sample in each branch will double to 18. Since the sampling rate of the branch signals is 45,000 samples per second, the sampling interval is 22.22 μs per sample. This means that 18 real multiplies will have to be performed every 22.22 μs. One channel will require 18/22.22 μs = 0.810 multiply per microsecond. For 800 channels, there will be 800 × 0.810 = 648 real multiplies per microsecond. Equivalently, the time required for one multiply is 1.5432 ns.

To accomplish this rate, 0.648 × 25 = 16.2 multipliers are required. Approximating the number of required multipliers to be 17, the estimated power needed to implement them is 17 × 150 mW = 2.55 W.

Computations for Preprocessing

In this block, each multiplier performs one complex multiply every 22.22 μs. This requires 0.045 complex multiply per microsecond for one channel. For 800 channels, the total number of multiplies is 800 × 0.045 = 36 complex multiplies per microsecond. One complex multiply requires four real multiplies. Thus, the preprocessing block will have a total of 144 real multiplies per microsecond. This corresponds to 0.144 real multiply per nanosecond (6.94 ns per multiply).

The number of multipliers required will be 0.144 × 25 = 3.6, which is approximately four multipliers. This results in a consumption of 4 × 150 mW = 0.6 W of power.

Computations for the Postprocessing

Since in this block only the sign change operation is performed, no multiplications are required.

Computations for GWHT and Correction Matrix

In order to utilize the Radix-2 or Radix-4 butterfly structure (without the twiddle factors) to implement the GWHT in hardware, a transform size of $NT = 1024$ is chosen. To represent a matrix size of 1024, the parameters m and n are chosen as $m = 4$ and $n = 5$ since these parameters give a simplified GWHT matrix. Based on these parameter values, the GWHT matrix has trivial coefficients (i.e., ± 1 or $\pm j$). Thus, a structured implementation of the GWHT will result in zero multiplications.

In Ref. 10, it is stated that each row of the correction matrix will have $(m - 1)NT/m$ entries that are zero-valued. With $m = 4$ and $NT = 1024$, each row of the correction matrix will have 256 nonzero complex coefficients. Based on this theoretical result, one channel will require 256 complex multiplies per sample. As an example, assume that only 10% of the total number of carriers are desired. Then the correction matrix will require 80 × 256 = 20,480 complex multiplies per sample. Thus 81,920 real multiplies per sample are needed, since one complex multiply requires four real multiplies. To accomplish this rate within a 22.22 μs time interval, 3.68 real multiplies per nanosecond are required. Therefore, a total of $3.68 \times 25 = 92$ multipliers should be used to accomplish the required multiplication rate. The power consumption for the 92 multipliers is 92×150 mW = 13.8 W.

The total power required to implement the proposed system for the situation where only 80 out of 800 carriers are present is obtained by adding the power requirements for the polyphase filtering, preprocessing, and the correction matrix functions $(2.55 + 0.6 + 13.8) = 16.95$ W. The GWHT and the postprocessor blocks are not considered since the power estimates for these two blocks will be minimal when compared to the rest of the blocks.

Comparison with DFT and FFT

For the example of 800 carriers, the implementation of the DFT in its direct form will require an 800×800 matrix. Each row of the DFT matrix will have 800 complex coefficients. If L carriers need to be processed by the DFT matrix, then the number of multiplications is $800 \times L$ complex multiplies per sample. Alternatively, if FFT (Radix-2 algorithm) is employed, the number of multiplications will remain constant with respect to L at $(NT/2) \log_2 NT$ complex multiplies. For 800 carriers it would be appropriate to choose a transform size of 1024. Thus, the computational rate required for the FFT is 5120 complex multiplies per sample. This rate is regardless of the number of channels desired.

To compare the proposed technique—approximation and subsequent correction—with both the DFT and the FFT approaches, only the computational rates for the correction matrix need to be compared. This is because the GWHT with trivial coefficients will always result in zero multiplies. Thus, by comparing the correction block with DFT, it can be seen that no matter how many carriers are needed, the proposed transform will always be computationally more efficient than DFT. The structure of the correction matrix requires $256 \times L$ complex multiplies per sample, which is always better than the $800 \times L$ complex multiplies per sample required by DFT.

When compared to FFT, it is observed that the proposed transform technique is efficient only when a small number of carriers need to be computed. Based on the theoretical structure, at $L = 20$, we have $20 \times 256 = 5120$ complex multiplies per sample, which is the same as that required for FFT. For any $L > 20$, according to the theoretical structure, the proposed transform technique will require more computations than FFT. A computer program was written to exploit the structure of the correction matrix, from which following observations are made for $NT = 1024$ ($m = 4$, $n = 5$):

4 rows will have 1 nonzero complex coefficient
12 rows will have 4 nonzero complex coefficients
48 rows will have 16 nonzero complex coefficients
192 rows will have 64 nonzero complex coefficients
768 rows will have 256 nonzero complex coefficients

However, out of 1024 rows, only the top 800 rows are needed, thus the bottom 224 rows of the 1024×1024 matrix are never accessed. Based on this, the correction matrix structure for the top 800 rows is as follows:

4 rows will have 1 nonzero complex coefficient
9 rows will have 4 nonzero complex coefficients
38 rows will have 16 nonzero complex coefficients
150 rows will have 64 nonzero complex coefficients
599 rows will have 256 nonzero complex coefficients

If we compare the observed structure of the correction matrix with FFT, then L can be as large as 121 out of 800 carriers. But, this efficiency is very much dependent on the location of the carriers of interest.

Conclusion

In this paper, the concept of separating few out of several carriers is discussed. A demultiplexing algorithm is proposed that would enable demultiplexing of only the carriers of interest from the wideband spectrum of the FDMA signal in a random and efficient manner. Based on the computational requirements, it is observed that the proposed technique of approximation and subsequent correction is efficient when only a small number of carriers need to be demultiplexed. When compared to the conventional fast transform algorithms (such as FFT), the efficiency of the proposed algorithm decreases as the number of carriers increase. It is found that if the number of carriers to be processed is less than 20 (out of 800), then this demultiplexing algorithm is superior to FFT. This conclusion suggests that for an Earth station application where the interest may be in only few carriers out of the hundreds it receives, the proposed demultiplexing algorithm is highly applicable.

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